where the following abbreviations are used:

$$a = a_1a_2, \quad b = b_1b_2, \quad A = a_2 - a_1, \quad B = b_2 - b_1,$$

$$C = a_1b_2 - a_2b_1, \quad D = a_1b_2 + a_2b_1, \quad L = C[ADm + (m+2)aB],$$

$$M = C[BDm + (m+2)bA], \quad N = a(C^2m + aB^2), \quad P = b(C^2m + bA^2),$$

$$Q = maAC, \quad R = mbBC, \quad S = (mC^2D + 2abAB).$$

If we look upon (8) as an algebraical equation in $p = dy/dx$ which has 4 roots p_1, p_2, p_3, p_4, these being functions of x and y, then we are led to the conclusion that through any point in the plane there are *four* directions satisfying the condition proposed. We have, therefore, *four* curves in question.

For the present purpose the equation (8) in p may be considered irreducible. Then, if a singular solution of this equation exists, it must simultaneously satisfy the equations

$$\varphi = 0, \quad \frac{\partial\varphi}{\partial p} = 0, \quad \frac{\partial\varphi}{\partial x} + p\frac{\partial\varphi}{\partial y} = 0.$$

The last equation is *identically zero* in the present case, and we have

$$\frac{d\varphi}{dp} = 4p^3Qx - 3p^2(C^2x^2 - Lx + Py + N) - 2p(2C^2xy + Mx - Ly - S) - (C^2y^2 + My - Rx + P) = 0. \tag{9}$$

Equations (8) and (9), therefore, represent, in parametric form, the singular solution of the differential equation (8), which is, then, the algebraic curve in question.

411. Proposed by C. N. SCHMALL, New York City.

$ABCD$ is a rectangle of known sides. BC being produced indefinitely, it is required to draw a straight line from A cutting CD and BC in X and Y, respectively, so that the intercept XY may be equal to a given straight line. (Unsolved in *Educational Times.*)

II. Solution by the Proposer.

We shall assume that the given rectangle is a square, the problem thus being a special case of problem 382, proposed by R. C. Archibald in the May (1911) number of the Monthly.

Construction: Along AB produced lay off AE equal to the *given length.* Draw ED. Prolong AD to K so that $DK = DE$. On AK as a diameter (centre O) describe a semi-circle cutting BC produced in Y. Draw AY cutting DC in X. Then AXY is the line required; *i. e.*, the intercept XY is of the given length.

Proof: Draw YH perpendicular to AK. Draw XK and YK. Then the right triangles ADX and YHK are equal in all respects (congruent). For the angles DAX and HYK are equal, and $DA = DC = HY$.

Hence

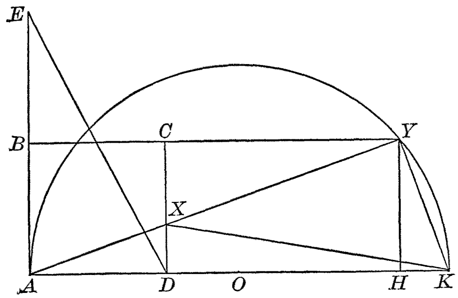
$$AX = YK. \tag{1}$$

Now

$$\overline{XY}^2 + \overline{YK}^2 = \overline{XK}^2 = \overline{XD}^2 + \overline{DK}^2 = \overline{XD}^2 + \overline{DE}^2 \text{ (by constn.)} \tag{2}$$

$$= \overline{XD}^2 + \overline{AD}^2 + \overline{AE}^2 = \overline{AX}^2 + \overline{AE}^2 = \overline{YK}^2 + \overline{AE}^2, \text{ by (1).}$$

Hence, $\overline{XY}^2 = \overline{AE}^2$ and, therefore, $XY = AE =$ the given length.



Also solved by J. Scheffer whose conclusion agrees with that of Professor Shively published in the April number.

H. C. Feemster sent in a solution of 409 too late for notice in April number.

MECHANICS.

250. Proposed by C. N. SCHMALL, New York City.

A smooth circular table in surrounded by a smooth vertical rim. A ball of elasticity e is projected from a point at the rim in a line making an angle θ with the radius through that point. Show that the ball will return to the starting point after the second impact if

$$\tan\theta = \sqrt{\frac{e^3}{e^2 + e + 1}}.$$

Solution by the Proposer.

Let ABC be the path of the ball and O the center of the table. Let the angles OAB, OBC, be θ, ϕ respectively. Then OBA and OCB are also θ, ϕ, respectively. Let OCA be ψ. Then if the ball returns to A after the second impact the angle CAO will also be ψ. The angles θ, ϕ, ψ, are the angles which the path of the ball makes with the radii passing through it initially and after the first and second impacts. We then have, by elementary principles of impact,

$$\cot\phi = e\cot\theta, \tag{1}$$

$$\cot\psi = e\cot\phi = e^2\cot\theta. \tag{2}$$

But $\theta + \phi + \psi = \pi/2$. Hence $\cot(\theta + \phi + \psi) = 0$.

That is,

$$\frac{\cot\theta\cot\phi\cot\psi - \cot\theta - \cot\phi - \cot\psi}{\cot\theta\cot\phi + \cot\phi\cot\psi + \cot\psi\cot\theta - 1} = 0.$$